NUMERICAL OPTIMIZATION OF MICROWAVE OSCILLATORS AND VCOs

Vittorio RIZZOLI (1), Alessandra COSTANZO (1), and Claudio CECCHETTI (2)

(1) Dipartimento di Elettronica, Informatica e Sistemistica, University of Bologna
(2) Fondazione Ugo Bordoni
Villa Griffone, 40044 Pontecchio Marconi, Bologna, ITALY

ABSTRACT

A new harmonic-balance optimization technique for single-frequency oscillators and broadband VCOs is introduced. The objective function is evaluated by a Newton-iteration based HB analysis, and an exact algorithm for the computation of the gradient is implemented. The optimizable performance in the VCO case includes the linearity of the tuning characteristic.

INTRODUCTION

The numerical optimization of forced nonlinear microwave circuits by the harmonic-balance (HB) method is well established and is currently available in general-purpose user-oriented CAD simulators. In its most usual implementation, this technique relies upon two nested numerical loops. The inner analysis loop computes the objective function by a Newton-iteration based HB circuit analysis, and simultaneously provides the basic numerical information required to find the gradient of the objective. The outer optimization loop minimizes the objective by some suitable nonlinear optimization strategy. The use of exact algorithms for the computation of the Jacobian matrix in the HB analysis [1] and of the gradient in the optimization [2], [3] ensures the speed and robustness of the numerical process. Many available benchmarks show that this approach provides an algorithmic efficiency comparable to the optimization of conventional linear circuits (e.g., [1]).

For autonomous circuits such as oscillators, an optimization technique of comparable efficiency is not available at present. This is due to the fact that the most commonly used approach to oscillator analysis relies upon the concept of continuation [4] in order to suppress the degenerate solution(s) of the HB equations for the autonomous case. According to this method, the autonomous circuit is reduced to a forced one by introducing in it a fictitious source (*probe*), and the solution is found by gradually reducing to zero the probe voltage or current. This technique is clearly not very well suited for use in conjunction with an optimization algorithm, because it requires a sequence of HB analyses for a single evaluation of the circuit performance.

In this paper we propose a novel approach to oscillator optimization, representing the logical extension to the autonomous case of the above-mentioned technique for forced circuits [1]. The analysis step is performed by a single *mixed-mode* Newton iteration [5], which provides the highest possible computational speed. Exact sensitivities with respect to the optimization variables are computed both for the usual network functions and for the tuning parameter(s). This allows the optimization not only of conventional free-running oscillators, but also of tunable oscillators such as VCOs specified over a frequency band. As in the analysis case, the elimination of the degenerate solution(s) is committed to an auxiliary algorithm [6], which is run for only a few iterations at the beginning of the numerical process, in order to provide the Newton-iteration based optimization with a suitable starting point. All this results in a numerical tool for the design of microwave oscillators and VCOs, both single-frequency and broadband, whose efficiency and generality of application are definitely superior to those of most presently available commercial CAD packages.

THE OPTIMIZATION ALGORITHM

Let us consider an autonomous nonlinear microwave circuit operating in a large-signal time-periodic electrical regime. A set **P** of real designable parameters is available in the circuit for optimization purposes. These optimization variables usually represent physical or electrical circuit parameters and/or bias voltages. The vector **P** must be found in such a way that the circuit supports an oscillatory steady state which in turn must satisfy a number of design goals, e.g., on output power, DC to RF conversion efficiency, and so on. It will be assumed that the performance specifications also include the fundamental frequency of oscillation, as it is usually the case in practice. The circuit state is described by a set of time-dependent state variables (SV), and thus, in the frequency domain, by the set of the SV harmonics. Since the system is autonomous, however, its electrical regime is invariant with respect to a shift of the time origin, so that the imaginary part (or the phase) of an arbitrary harmonic chosen as reference may be set to zero. In order to restore the correct number of degrees of freedom, such imaginary part is replaced by a suitable *tuning parameter* T [5], usually a circuit parameter or a bias voltage. Thus an oscillatory state of the autonomous system is described by a *mixed-mode* state vector **X** [5], containing the real and imaginary parts of the SV harmonics (except for the imaginary part of the reference harmonic) and the tuning parameter.

According to the piecewise HB technique, the circuit is subdivided into a linear and a nonlinear subnetwork interconnected through a number of common ports. The oscillatory steady states of the circuit are then defined by the solutions of a nonlinear algebraic system of the form

$$\mathbf{E}(\mathbf{X}, \mathbf{P}) = \mathbf{0} \qquad (1)$$

where **E** is the set of the real and imaginary parts of all harmonic-balance errors (i.e., at all spectral lines to be taken into account in the analysis, and all common ports). For any given set **P** of optimization variables, (1) may be solved with respect to the state vector **X** by a Newton iteration [5]. The Jacobian of **E** with respect to **X** is computed by the exact algorithms discussed in [1]. This step simultaneously determines the tuning parameter and the oscillatory steady state, from which the performance indexes and the objective function may be derived.

The optimization process can be viewed as the search for

CH3277-1/93/0000-0629$01.00

a set **P** of design variables for which the specifications are satisfied in the best possible way, subject to the constraint that the state lies on the manifold $\mathbf{M} \equiv [\mathbf{X} = \mathbf{X}(\mathbf{P})]$ implicitly defined by (1). The search process is reduced to the minimization of an objective function defined in a conventional way. A generic design goal is first expressed in the form

$$F^{(i)}_{min} \leq F^{(i)}(\mathbf{X}, \mathbf{P}) \tag{2}$$

where $F^{(i)}(\mathbf{X}, \mathbf{P})$ is the network function to be specified. For a two-sided least-pth (ℓ_p) objective [7], the inequality (2) is associated with the error function

$$E^{(i)}(\mathbf{X}, \mathbf{P}) = w^{(i)} \cdot \left[F^{(i)}_{min} - F^{(i)}(\mathbf{X}, \mathbf{P})\right] \tag{3}$$

where $w^{(i)}$ is a positive weight. Then, if E_{max} is the maximum error (in the algebraic sense), the objective function is given by [7]

$$F_{OB}(\mathbf{P}) = \begin{cases} \text{if } E_{max} \geq 0: \\ \left\{ \sum_i^{+} \left[E^{(i)}(\mathbf{X}(\mathbf{P}), \mathbf{P}) \right]^p \right\}^{1/p} \\ \text{if } E_{max} < 0: \\ -\left\{ \sum_i \left[-E^{(i)}(\mathbf{X}(\mathbf{P}), \mathbf{P}) \right]^{-p} \right\}^{-1/p} \end{cases} \tag{4}$$

where the superscript $^+$ indicates that the summation is extended to positive errors only, and $p > 1$. To carry out the design, the objective (4) is minimized by any suitable algorithm for constrained optimization. For best efficiency and robustness, gradient-based algorithms are preferred [2], and the gradient is computed by the exact technique developed in [1].

SINGLE-FREQUENCY AND BROADBAND OPTIMIZATION

The typical oscillator design problem is a single-frequency problem. This means that the required fundamental (angular) frequency of oscillation, ω_0, is a priori assigned as a design specification, and the spectrum to be considered in the HB analysis is simply a finite set of harmonics of ω_0. In this case, according to the above discussion, the tuning parameter T is treated as an unconstrained degree of freedom. The specific task of the tuning parameter is to compensate the changes of the design variables at each iteration, in such a way as to restore the prescribed frequency of oscillation. Thus a natural choice for T is a frequency-determining parameter of the linear subnetwork, such as a reactive component of the feedback branch. At each iteration the tuning parameter is found together with the electrical regime by a single Newton iteration [5]. This results in a major speed advantage of the new optimization method proposed in this paper with respect to previously available techniques.

A broadband optimization problem may occur for a tunable oscillator. In this case the oscillator performance is simultaneously specified for a number of discrete values of the fundamental frequency of oscillation, say $\omega_1, \omega_2, ..., \omega_R$, suitably located across the band of interest. In turn, an independent state vector $\mathbf{X}_r$ is associated with each ω_r, so that the set of problem unknowns consists of $\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_R$, and **P**. The discussion of the previous section is still formally valid for the broadband case. The analysis step now consists of a sequence of R independent mixed-mode Newton iterations, by which the oscillatory regime at each fundamental and the associated value of the tuning parameter, $T(\omega_r)$, are determined. The objective function is still computed by (3) and (4), where the index i now spans all the constraints and all the fundamental frequencies of interest. The tuning parameter may still represent a circuit variable (e.g., mechanical tuning of a cavity oscillator), but most often coincides with a DC voltage, such as a varactor bias in a VCO. This situation is particularly favourable since the mixed-mode Newton iteration then becomes especially fast [5].

A peculiar aspect of the broadband optimization of a tunable oscillator is that the design goals may involve the tuning parameter. A simple but important example is the linearity requirement for the tuning characteristic of a VCO, which may be stated in the form

$$A\,\omega_r + B - \varepsilon \leq T(\omega_r) \leq A\,\omega_r + B + \varepsilon \qquad 1 \leq r \leq R \tag{5}$$

where A, B are constants, ε is the allowed tolerance, and $T(\omega)$ is chosen as a varactor bias voltage. In some cases A, B can be a priori assigned. However, in most practical situations they can be regarded as free parameters, in the sense that any tuning characteristic satisfying the constraint (5) with respect to an *arbitrary* straight line is acceptable. If this is the case, the parameters A, B are updated at the beginning of each iteration by finding the straight line that provides the best fit to the actual tuning curve. A and B are always suitably constrained in order to prevent the DC voltage across the varactor junction from exceeding the breakdown voltage.

An important feature of the optimization process is that it allows the computation of the exact derivatives of the tuning parameter with respect to the optimization variables. Let $T = \mathbf{S}\,\mathbf{X}$ where $\mathbf{S} \triangleq [0 \ ... \ 1 \ ... \ 0 \ \ 0]$, the nonzero entry corresponding to the position of T in the vector **X**. Also let the derivatives be denoted by the symbol D when they are taken on the manifold **M**. Differentiating (1) then yields

$$\frac{DT}{D\mathbf{P}} = \mathbf{S}\frac{D\mathbf{X}}{D\mathbf{P}} = -\mathbf{S}\,\mathbf{J}^{-1}\left.\frac{\partial \mathbf{E}}{\partial \mathbf{P}}\right|_{\mathbf{X}=\text{const.}} \tag{6}$$

where **J** is the Jacobian of **E** with respect to **X**. The essential point here is that $\mathbf{J}^{-1}$ is immediately available after performing an HB analysis by the Newton iteration. For the rest, the specification (5) may be formally treated as any other design goal of the general form (2).

THE DEGENERATE SOLUTION PROBLEM

Since an autonomous circuit only contains DC sources, the harmonic-balance system (1) always admits at least one *degenerate* solution, for which the only nonzero harmonics are DC components. For the oscillator analysis problem, a general technique for the elimination of such solution(s) has been discussed in [6]. In the optimization case, two kinds of difficulties related with the existence of degenerate solutions may be encountered: i), the initial value of **P** may lie outside the region of the parameter space where oscillatory states exist; ii), during the optimization the Newton iteration may converge to a degenerate solution. If in a neighborhood of a point **P** no oscillatory states are found, **P** is interpreted as a local minimum by the gradient-based optimizer, and the optimization may collapse. There are obviously many possible solutions to these problems. One approach that has proven very effective in practice is to implement a backup algorithm based on a different definition of the objective function, namely

$$F_{OB}(\mathbf{X}, \mathbf{P}) = \begin{cases} \text{if } E_{max} \geq 0: \\ \left\{ \sum_{i}^{+} \left[E^{(i)}(\mathbf{X}, \mathbf{P}) \right]^{p} + \| w_E \, \mathbf{E}(\mathbf{X}, \mathbf{P}) \|^{p} \right\}^{1/p} \\ \\ \text{if } E_{max} < 0: \\ w_E \, \| \mathbf{E}(\mathbf{X}, \mathbf{P}) \| \end{cases} \tag{7}$$

where $\| \bullet \|$ denotes the Euclidean norm and w_E is a positive weight. When (7) is used, at a generic iteration the vectors **X**, **P** do not represent a solution of the harmonic-balance system (1) unless $F_{OB} = 0$. Thus the optimization based on (7) is not sensitive to degenerate solutions, and can provide a smooth specification-driven transition from any initial state (including zero) to an oscillatory state, through a sequence of physically meaningless, but nevertheless acceptable iterations. In practice, since the algorithm based on (3), (4) is much more efficient, (7) is activated only when a critical situation such as i) or ii) is encountered. Experience shows that in such cases, only a few iterations based on (7) are sufficient to move **P** to a point in the parameter space from where the Newton-iteration based algorithm can be safely (re)started. This procedure is automatically handled by the program in a way transparent to the user.

AN EXAMPLE OF APPLICATION

Let us consider the varactor-tuned oscillator whose topology is schematically illustrated in fig. 1. The oscillator has to be designed as a VCO tunable over an 800 MHz band centered around 4.5 GHz, with a maximum deviation from linearity of ±40 MHz across this band. A minimum output power of 12 dBm and a minimum drain efficiency of 15% are prescribed throughout the tuning band. The designable parameters are the lengths of the microstrip lines shown in fig. 1. All characteristic impedances are arbitrarily set to 50 Ω. The output stub tuner is intended to provide a broadband matching of the drain to the load resistor. On the other hand, the purpose of the multiple-stub reactance-compensating network connected to the gate is to tailor the frequency dependence of the feedback reactance in such a way as to linearize the tuning characteristic. At the starting point all lengths are set to λ/4 at center band (λ/2 for the open stubs). 8 harmonics including the fundamental are taken into account in all HB analyses.

A 160 μm FET biased at 5V and 25 mA is chosen as the active device. The varactor has a zero-bias depletion-layer capacitance of 1.7 pF and a breakdown voltage of -25 V. A single-frequency optimization is first carried out with a 1 dB margin on the nominal specification on output power. The varactor bias voltage is used as the tuning parameter T, with a starting value of -10 V. Since at the starting point the circuit does not oscillate, the optimization is carried out by 30 iterations based on (7) followed by 11 iterations based on (3), (4). The CPU times are 18 seconds and 25 seconds, respectively, on a SUN SPARCstation 2. The final point meets the specifications, and exhibits the tuning characteristic shown in fig. 2. Note that the shaded part of the tuning characteristic is not physically meaningful because in this region the DC voltage across the varactor exceeds the breakdown voltage. This situation is acceptable provided that the problem be eliminated at the end of the broadband optimization. Fig. 3 shows the frequency deviation from the best-fit linear characteristic. The maximum deviation from linearity is found to be ±196 MHz. The output power is plotted in fig. 4 and the drain efficiency in fig. 5.

Starting from the results of the previous step, a broadband optimization is then carried out with nominal specifications (7 optimization variables). In order to compute the broadband objective function, R = 9 values of the fundamental frequency of oscillation, uniformly distributed across the band of interest, are simultaneously taken into account. The optimization converges in 52 iterations based on (3), (4), and does not require any iteration based on (7). The CPU time is 890 seconds on a SUN SPARCstation 2. The final point meets all specifications, and its tuning characteristic is shown in fig. 2. In particular, the DC voltage across the varactor is always well below the breakdown voltage. Fig. 3 shows the frequency deviation from the best-fit linear characteristic. The maximum deviation from linearity is found to be ±37 MHz. The output power is again plotted in fig. 4 and the drain efficiency in fig. 5.

ACKNOWLEDGMENTS

This work was partly sponsored by the Italian Space Agency (ASI) and by the Istituto Superiore delle Poste e delle Telecomunicazioni (ISPT).

REFERENCES

[1] V. Rizzoli *et al.*, "State-of-the-art harmonic-balance simulation of forced nonlinear microwave circuits by the piecewise technique", *IEEE Trans. Microwave Theory Tech.*, Vol. 40, Jan. 1992, pp. 12-28.

[2] J. W. Bandler, Q. J. Zhang, and R. M. Biernacki, "Practical, high speed gradient computation for harmonic balance simulators", *1989 IEEE MTT-S Int. Microwave Symp. Digest* (Long Beach), June 1989, pp. 363-366.

[3] V. Rizzoli, A. Costanzo, and C. Cecchetti, "Numerical optimization of broadband nonlinear microwave circuits", *1990 IEEE MTT-S Int. Microwave Symp. Digest* (Dallas), May 1990, pp. 335-338.

[4] D. Hente and R. H. Jansen, "Frequency-domain continuation method for the analysis and stability investigation of nonlinear microwave circuits", *Proc. Inst. Elec. Eng.*, pt. H, Vol. 133, Oct. 1986, pp. 351-362.

[5] V. Rizzoli and A. Neri, "Harmonic-balance analysis of multitone autonomous nonlinear microwave circuits", *1991 IEEE MTT-S Int. Microwave Symp. Digest* (Boston), June 1991, pp. 107-110.

[6] V. Rizzoli, A. Costanzo, and A. Neri, "Harmonic-balance analysis of microwave oscillators with automatic suppression of degenerate solution", *Electron. Lett.*, Vol. 28, Jan. 1992, pp. 256-257.

[7] C. Charalambous, "Nonlinear least pth optimization and nonlinear programming", *Mathematical Programming*, Vol. 12, 1977, pp. 195-225.

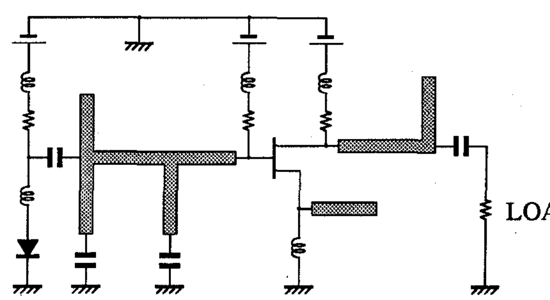

Fig. 1 - Schematic topology of a VCO

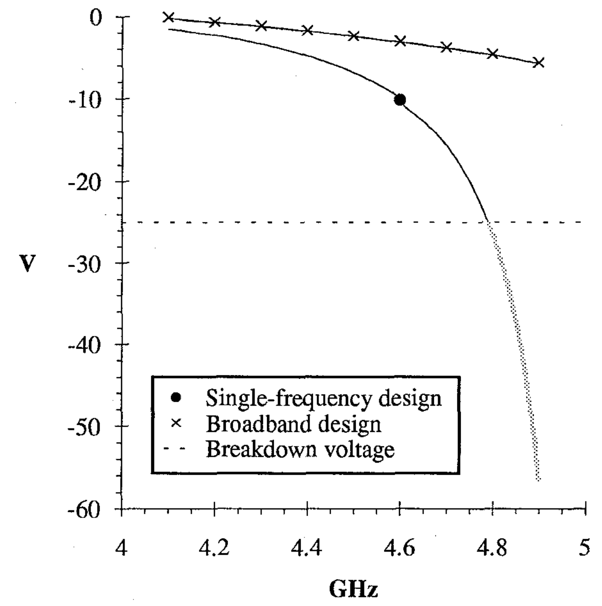

Fig. 2 - Varactor tuning voltage vs. frequency of oscillation

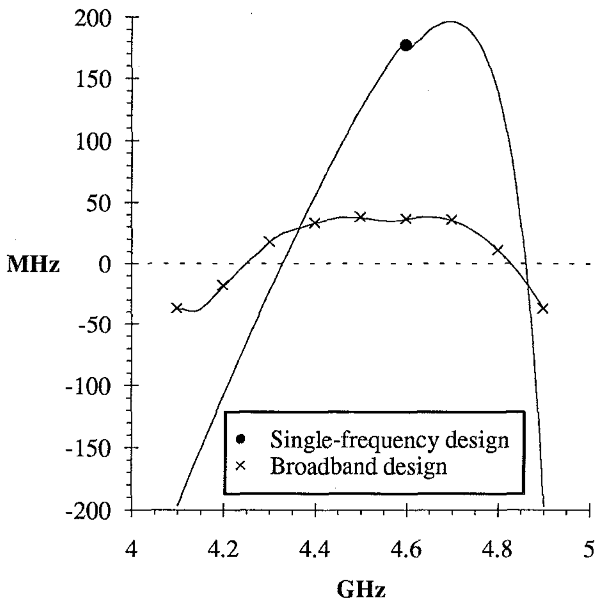

Fig. 3 - Frequency deviation from the best-fit straight line vs. frequency of oscillation

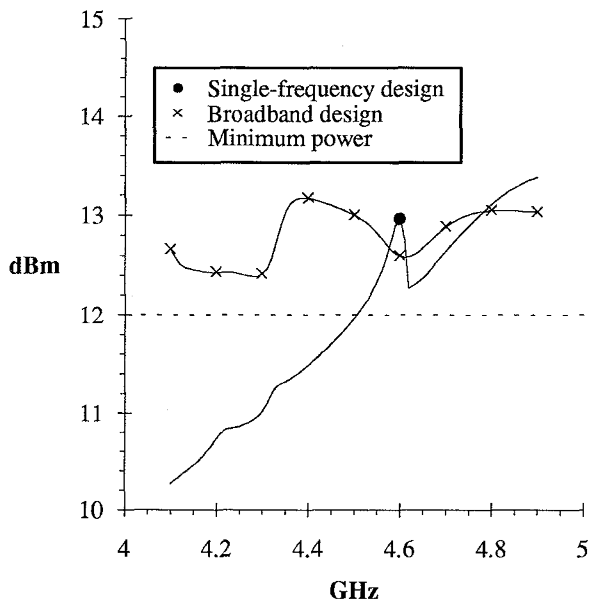

Fig. 4 - Output power vs. frequency of oscillation

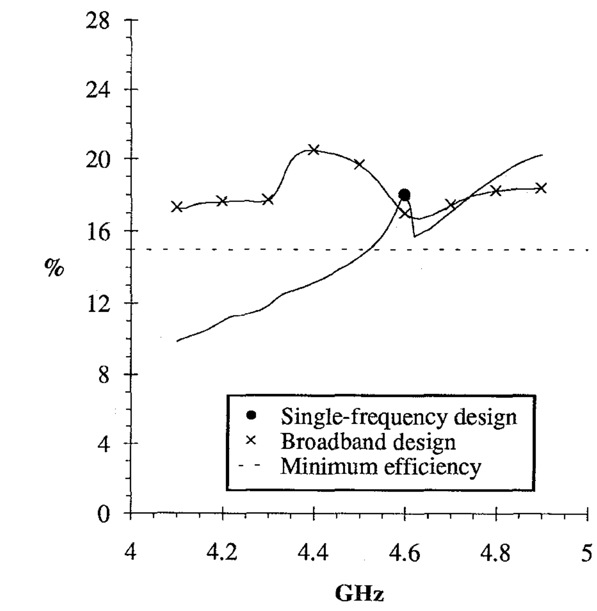

Fig. 5 - DC to RF conversion efficiency vs. frequency of oscillation